Entangling two superconducting *LC* coherent modes via a superconducting flux qubit

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Based on a pure solid-state device consisting of two superconducting *LC* circuits coupled to a superconducting flux qubit, we propose in this paper that the maximally entangled coherent states of the two *LC* modes can be generated for arbitrary coherent states through flux qubit controls.

I. INTRODUCTION

Quantum entanglement is not only of interests in the fundamentals of quantum mechanics concerning the EPR paradox[1] but also serves as an indispensable resource for quantum information processing.[2] Many discrete entangled states in terms of polarized photons, atoms, trapped ions, and electrons in nanostructures have been experimentally demonstrated. However their practical applications suffer from single-particle decoherence severely. Therefore, increasing attention has been paid to generating macroscopic entangled states[3–6] due to their robustness against single-particle decoherence. The entangled coherent states is one of the most important ingredients of quantum information processing using coherent states.[7] Creating entangled coherent states, initially proposed by Sanders in quantum optics,[8] have been extensively explored in many other systems, such as trapped ions,[9] microwave cavity QED,[10] Boson-Einstein Condensation (BEC) system,[11] as well as the nanomechanical systems[12] but not yet realized experimentally.

Motivated by the recent experiments on strong coupling between superconducting *LC* resonators and superconducting flux qubits,[13,14] we propose in this paper a pure electronic (solid-state) device for generating entangled coherent states of two superconducting *LC* modes through flux qubit controls. Using superconducting qubits coupled with a *LC* resonator (as a quantum bus) to generate superconducting qubit couplings, to build two-qubit entanglement, and to implement two-qubit logic gates have been extensively studied for quantum information processing in the past years.[15–25] Here, we shall design an alternative superconducting circuits that using the measurement of superconducting flux qubit states to generate the maximum entanglement states of the two *LC* coherent modes for quantum communication. The scheme of generating entanglement states of distant systems through measurement was indeed proposed a decade ago.[26] However, *LC* circuits are building blocks of all the electronic information and communication devices used today, this entangled *LC* coherent mode generator could be very promising for practical realization of quantum communication and quantum information processing.

II. SYSTEM SETUP

The device we design here consists of two superconducting *LC* circuits strongly coupled to a superconducting flux qubit. Figure 1(a) is a schematic setup of our superconducting circuits. The central circuit is a superconducting flux qubit which is coupled to two superconducting *LC* circuits through mutual inductance. The qubit is enclosed by a superconducting quantum interference device (SQUID) as a qubit measurement device. Coherent control of the qubit is achieved via two microwave control lines (I_1, I_2). Symmetric circuits are designed to suppress excitation of the SQUID and to protect the two *LC* oscillators from the unwanted influence of the qubit controlling pulses.

Both the superconducting *LC* circuits and the flux qubit can be fabricated on a chip down to the micrometer scale. The superconducting *LC* circuit is an ideal harmonic oscillator verified experimentally,[13] and the two levels of the superconducting flux qubit comprise of the clockwise and counterclockwise persistent-current states $|0\rangle$ and $|1\rangle$.[27,28] The latter is made of a superconducting loop interrupted by three Josephson junctions[27] in which two junctions have the same Josephson coupling energy E_J, and the third junction [placed by a SQUID in Fig. 1(a)] has the coupling energy smaller than that of the other two junctions by a factor α with $0.5 < \alpha < 1$. The interaction of the flux qubit and two *LC* circuits can be controlled by the external microwave control lines. The geometrical structure of the *LC* circuit is adjustable so that the strong coupling can be achieved.[13] The flux qubit is also tunable and has the advantage of long-decoherence time. These advantages decrease the difficulty of the experiment and increase the feasibility.

Preparing the flux qubit in a superposition of the states $|0\rangle$ and $|1\rangle$ initially, we are able to drive the qubit and the two *LC* modes into a tripartite entanglement [see Fig. 1(b) and Eq. (4)]. Measuring the qubit state with an enclosed dc SQUID, which is inductively coupled to the qubit[28,29] as shown in Fig. 1(a), will generate the entangled *LC* coherent modes. This is the procedure of entangling two superconducting *LC* coherent modes through flux qubit controls. As schematically depicted in Fig. 1(a), the qubit detector consists of a ring interrupted by two Josephson junctions. This SQUID is connected in such a way that the current can be injected through the parallel junctions. The switching current of the detector is sensitive to the flux produced by the current of the flux qubit. The readout of the qubit state is performed by applying a pulse sequence to the SQUID, as shown in Fig. 1(b), and recording whether the SQUID had been switched to a finite voltage (V_g) or remained in the zero voltage.
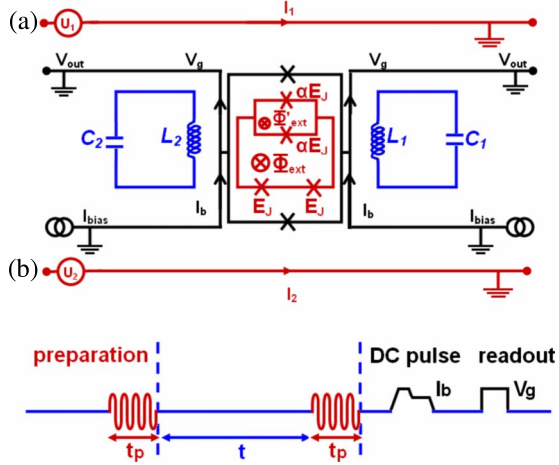
FIG. 1. (Color online) (a) Schematic diagram of a pure electronic device for entangling two LC modes through a flux qubit. The four junctions flux qubit is in the inner loop and is enclosed by a dc-SQUID detector (with two Josephson junctions). The two microwave lines modulate the flux in the qubit loop and control the parameters Δ and ε. The qubit state are read out by applying a current pulse I_b and then recording the voltage state of the SQUID. (b) Signals involved in quantum state manipulation and measurement. First, microwave pulses are applied to the qubit for state preparation. After the last microwave pulse, a readout current pulse I_b is injecting to the dc SQUID. The height and the length of the pulse are adjusted to give the best discrimination between the ground and the excited states. Finally, measuring the voltage state of the dc SQUID in which the voltage state of the dc SQUID depends on the switching probability of the energy eigenstates.

III. ENTANGLING TWO LC COHERENT MODES

Explicitly, the Hamiltonian of the total system can be described by[13]

$$H = \sum_{i=1}^{2} \hbar\omega_i a_i^\dagger a_i - \hbar\left(\frac{\varepsilon}{2}\sigma_z + \frac{\Delta}{2}\sigma_x\right) + \sum_{i=1}^{2}\hbar\lambda_i(a_i^\dagger + a_i)\sigma_z, \tag{1}$$

where $a_i^\dagger(a_i), i=1,2$ is the plasmon creation (annihilation) operator of the two LC oscillators, the corresponding resonance frequency ω_i is determined by the respective capacitance C_i and the inductance L_i; $\omega_i = \frac{1}{\sqrt{L_iC_i}}$ which is on the order of tens of gigahertz for a micrometer scale LC circuit. The operators σ_z and σ_x are the usual Pauli matrices describing the superconducting flux qubit. The energy splitting of the qubit is given by $\hbar\varepsilon = 2I_p(\Phi_{ext} - \frac{\Phi_0}{2})$ in which $I_p(0.3–0.5~\mu A)$ is the persistent current in the qubit, Φ_{ext} is the external magnetic flux applied in the superconducting loop, and $\Phi_0 = \frac{h}{2e}$ is the flux quantum. Δ is an effective tunneling amplitude describing qubit state flip, which depends on E_J.[30] The Josephson energy E_J, in turn, can be controlled when the third junction is replaced by a SQUID, as shown in Fig. 1(a), introducing the flux Φ'_{ext} as another control parameter.[27] These two external magnetic flux Φ_{ext} and Φ'_{ext} can be suddenly switched by two resonant microwave lines I_1 and I_2 for a finite time (approximate tens of picosecond) to

manipulate the two parameters, ε and Δ, respectively.[31] The LC circuits couple to the flux qubit via the mutual inductance with the coupling constant $\lambda_i = M_i I_p \sqrt{\frac{\omega_i}{2\hbar L_i}}$, where M_i ($\sim$pF) is the mutual inductance between the LC circuits and the flux qubit.[13,32] As we can see, the qubit energy splitting ε and the LC-qubit coupling λ_i are related through the persistent current I_p while the qubit flip amplitude Δ can be controlled independently through the additional external flux Φ'_{ext}.

The manipulating and measuring signal sequences on the flux qubit are shown in Fig. 1(b). First let the LC circuits be prepared in their ground states and the flux qubit in the state $|0\rangle$, the state of the total system at $t=0$ can then be written as $|\Psi(0)\rangle = |0\rangle|0_10_2\rangle$, where the subscripts 1 and 2 denote the two LC circuits. The qubit localized in $|0\rangle$ at $t=0$ also implies that the qubit flip amplitude Δ is initially adjusted to be much smaller than ε and λ_i, i.e., $\Delta_0 \ll \varepsilon, \lambda_i$. Then applying a pulse nonadiabatically [denoted by $P(t_p)$] to modulate the two control lines I_1 and I_2 such that Φ_{ext} is kept almost no change but Φ'_{ext} is changed dramatically. Since the LC resonators couple to the flux qubit through the σ_z component in our device,[13,33] the LC-qubit coupling λ_i is only sensitive to the change in the energy splitting ε of the qubit [also see the explicit expressions given after Eq. (1)]. This allows us to keep the parameters ε, λ_i almost as a constant but adjust the qubit flip amplitude Δ quickly to a large value to reach a condition $\Delta \gg \varepsilon, \lambda$ through the change in Φ'_{ext}. As a result, this pulse drives the flux qubit into the degeneracy point within a duration $t_p = \frac{\pi}{2\Delta}$ without disturbing the LC resonator states too much. Accordingly, the state $|\Psi(0)\rangle$ evolves to

$$|\Psi(t_p)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)|0_10_2\rangle. \tag{2}$$

In fact, it has been shown recently that for a similar system,[14,34,35] the qubit flip amplitude Δ can be rapidly increased while the qubit energy splitting ε and the qubit-LC coupling λ_i vanish rather abruptly through the non-adiabatic control of the flux (see explicitly Fig. 3 in Ref. 35).

After the first pulse, the parameters return to the initial values $\Delta \rightarrow \Delta_0 \approx 0$, namely, the σ_x term in the Hamiltonian now contributes little effect on the subsequent evolution of the qubit. Then let the system evolve lasting a period of time t, the resulted state is given by

$$|\Psi(t_p+t)\rangle = \frac{1}{\sqrt{2}}[e^{-i\varepsilon t/2}|0\rangle|\kappa_1(t)\kappa_2(t)\rangle$$
$$+ ie^{i\varepsilon t/2}|1\rangle|-\kappa_1(t)-\kappa_2(t)\rangle], \tag{3}$$

where $|\kappa_i(t)\rangle \equiv e^{\kappa_i(t)a_i^\dagger - \kappa_i^*(t)a_i}|0_i\rangle$ is a coherent state characterized by the complex variable $\kappa_i(t) = \frac{\lambda_i}{\omega_i}(1-e^{-i\omega_i t})$. Equation (3) is a tripartite entangled state of one qubit with two coherent LC modes. We can apply the same pulse $P(t_p)$ to the flux qubit again [see Fig. 1(b)], the state of Eq. (3) is driven to

$$|\Psi(t_p + t + t_p)\rangle = \frac{1}{2} e^{-i\varepsilon t/2} \{|0\rangle [|\kappa_1'(t)\kappa_2'(t)\rangle - e^{i\varepsilon t}|$$
$$- \kappa_1'(t) - \kappa_2'(t)\rangle] + i|1\rangle [|\kappa_1'(t)\kappa_2'(t)\rangle$$
$$+ e^{i\varepsilon t}| - \kappa_1'(t) - \kappa_2'(t)\rangle]\}, \qquad (4)$$

where $\kappa_i'(t) = \kappa_i(t) e^{-i\omega_i t_p}$. We now measure the flux qubit in the σ_z basis, i.e., the natural computational basis $\{|0\rangle, |1\rangle\}$ which is indeed the energy eigenstate basis in the present case since $\Delta \to \Delta_0 \ll \varepsilon$ after the second pulse. As a result, the two *LC* modes collapse into the state,

$$|\psi_+\rangle_{12} = \frac{1}{\sqrt{2}}[|\kappa_1'(t)\kappa_2'(t)\rangle + e^{i\varepsilon t}| - \kappa_1'(t) - \kappa_2'(t)\rangle] \qquad (5)$$

if the qubit is measured with the result 1 or

$$|\psi_-\rangle_{12} = \frac{1}{\sqrt{2}}([|\kappa_1'(t)\kappa_2'(t)\rangle - e^{i\varepsilon t}| - \kappa_1'(t) - \kappa_2'(t)\rangle] \qquad (6)$$

if the measured result is 0. Each outcome has a probability of 50% to occur. Equations (5) and (6) are two entangled coherent states of the two superconducting *LC* circuits we propose to generate.

In practice, we are more interested in the case of the two superconducting *LC* circuits being symmetric in geometry for protecting the two *LC* oscillators from the unwanted influence of the qubit controlling pulses. Thus we have $\omega_1 = \omega_2 \equiv \omega$ and $\lambda_1 = \lambda_2 \equiv \lambda$. Let the system evolve for a period of time $t = \frac{\pi}{\omega}$ between the two pulses, we obtain the following standard form of the entangled two *LC* coherent states:

$$|\phi_\pm\rangle_{12} = \frac{1}{\sqrt{2}}[|2\kappa_0 2\kappa_0\rangle \pm e^{i\varphi}| - 2\kappa_0 - 2\kappa_0\rangle] \qquad (7)$$

with $\kappa_0 = \frac{\lambda}{\omega} e^{-i\omega t_p}$ and $\varphi = \pi \frac{\varepsilon}{\omega}$. Using the concept of concurrence for bipartite entangled nonorthogonal states,[9,36] it is easy to show that the concurrence for $|\phi_\pm\rangle_{12}$ is given by

$$C_{\phi_\pm} = \frac{1 - e^{-16|\kappa_0|^2}}{1 \pm e^{-16|\kappa_0|^2} \cos \varphi}. \qquad (8)$$

If the rate $|\kappa_0| = \frac{\lambda}{\omega} \gtrsim 0.5$, the exponential factor $e^{-16\kappa_0^2} \ll 1$. Then we have $C_{\phi_\pm} \simeq 1$, namely, $|\phi_\pm\rangle_{12}$ are nearly maximally entangled even though the average boson number $(=4|\kappa_0|^2)$ in the coherent state $|2\kappa_0\rangle$ is a small number. By well-designed circuits, one can let the ratio of coupling constant to the resonance frequency near to 1, i.e., $\kappa_0 \simeq 1$, then $|\langle -2\kappa_0|2\kappa_0\rangle|^2 = e^{-16|\kappa_0|^2} \simeq 10^{-7} \simeq 0$, namely, the two coherent states $|-2\kappa_0\rangle$ and $|2\kappa_0\rangle$ in the entangled state [Eq. (7)] can be nearly orthogonal.

However, if the average boson number is too small, the coherent states $|\kappa_i(t)\rangle$ are not truly macroscopic states such that the robustness against decoherence for the corresponding entanglement states could be faded. This weakness can be overcome by preparing the two *LC* circuits initially in two coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ while the flux qubit is still in the ground state $|0\rangle$. The initial state of the total system becomes $|\Psi'(0)\rangle = |0\rangle|\alpha_1\alpha_2\rangle$. Similarly using the pulse $P(t_p)$ to rotate the qubit state, $|\Psi'(t_p)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)|\alpha_1'\alpha_2'\rangle$, where α_i'

$= \alpha_i e^{-i\omega_i t_p}$. Then let the system evolve for a period of time t, the resulting state of the total system is

$$|\Psi'(t_p + t)\rangle = \frac{1}{\sqrt{2}}[e^{-i\theta}|0\rangle|\beta_{1+}(t)\beta_{2+}(t)\rangle + ie^{i\theta}|1\rangle|\beta_{1-}(t)\beta_{2-}(t)\rangle]. \qquad (9)$$

Here we have defined $|\beta_{i\pm}(t)\rangle \equiv |\alpha_i' e^{-i\omega_i t} \pm \kappa_i(t)\rangle$ and $\theta = \frac{\varepsilon}{2} t + i(\delta_1 + \delta_2)$ with $\delta_i \equiv \frac{\lambda_i}{2\omega_i}[(e^{i\omega_i t} - 1)\alpha_i'^* + (1 - e^{-i\omega_i t})\alpha_i']$. Again we can measure the flux qubit in the σ_z basis after reapplying the pulse $P(t_p)$ to the qubit, which results in the following entangled coherent states:

$$|\psi_\pm'\rangle_{12} = \frac{1}{\sqrt{2}}[|\beta_{1+}'(t)\beta_{2+}'(t)\rangle \pm e^{i2\theta}|\beta_{1-}'(t)\beta_{2-}'(t)\rangle], \qquad (10)$$

where $\beta_{i\pm}' = \beta_{i\pm} e^{-i\omega_i t_p}$. Now, the coherent states $|\beta_{i\pm}'(t)\rangle$ can be a very macroscopic state, depending on the initial voltages applied to the two *LC* circuits for generating the initial two coherent states $|\alpha_i\rangle$. While the entanglement measures of $|\psi_\pm'\rangle_{12}$ are almost the same as that of $|\psi_\pm\rangle_{12}$. To be specific, we consider the symmetric *LC* circuits again with $\alpha_1 = \alpha_2 \equiv \alpha$ and take $t = \frac{\pi}{\omega}$, the concurrence for $|\psi_\pm'\rangle_{12}$ is given by

$$C_{\psi_\pm'} = \frac{1 - e^{-16|\kappa_0|^2}}{1 \pm e^{-16|\kappa_0|^2} \cos(\varphi - 16|\kappa_0|\text{Im } \alpha')}. \qquad (11)$$

It shows that for a given $|\kappa_0| = \frac{\lambda}{\omega}$ such that $e^{-16|\kappa_0|^2} \ll 1$, we can always have $C_{\psi_\pm'} \simeq 1$ regardless of the value of $\alpha' = \alpha e^{i\omega t_p}$. In other words, the device we proposed here can generate maximally entangled states for arbitrary coherent states with arbitrary large oscillating amplitudes.

IV. DECOHERENCE ANALYSIS AND CONCLUSION

We have shown how to entangle two *LC* coherent modes through a superconducting flux qubit. To make the device feasible, we should also analyze various possible decoherence effect to the system. In solid-state systems decoherence comes from many redundant degrees of freedom that interact with the device. The noise may due to the emission from the superconducting *LC* circuits and the flux qubit, and from the control and detect of the qubit state. (i) In fact, the decoherence of the entangled coherent state due to the photoloss has been analyzed in detail recently by one of us in Ref. 37. (ii) Recent experiments demonstrated that the relaxation and dephasing times of the flux qubit are greater than $0.1 \ \mu s$,[14,25,38,39] longer enough for qubit operations which is on the order of tens of picoseconds, estimated from $t_p = \frac{\pi}{2\Delta} \sim 40$ ps for $\Delta \simeq 40$ GHz.[30] (iii) The SQUID may be inductively coupled to the two *LC* oscillators. But from the estimation of the Johnson-Nyquist noise in the bias circuit, it has been shown that this contribution is several orders of magnitude weaker.[13,40] (iv) The symmetric design of the *LC* as well as the dc-SQUID circuits has effectively suppressed the noise induced by qubit operations.[13,25,28,30] Put all these decoherence effects together, the estimated decoherence times from the different source are much longer than the typical time scale [the pulse time $t_p \sim 40$ ps and evolving time t

$= \frac{\pi}{\omega} \sim 0.1$ ns for $\omega \simeq 40$ GHz (Ref. 13)] of the system for producing entangled coherent states, which makes the system more practical.

In conclusion, we proposed a pure electronic (solid-state) device consisting of two superconducting LC modes coupled with a superconducting flux qubit. We showed that entangled coherent states of the two LC modes can be generated through the flux qubit controls. With the well-designed superconducting circuits one can achieve a strong coupling between the flux qubit and the LC circuits,[13,14] and the adjustable physical parameters gives extra degrees of freedom to generate the maximally entangled states for arbitrary coherent states. Beside being of the fundamental interest, the robust, macroscopic entanglement of two LC coherent modes described here is expected to be useful and powerful in quantum information processing. Such an entangled coherent state generating device is promising in practical applications since LC circuits are the building blocks of the information technology. Once the entanglement coherent states of the two LC modes can be experimentally realized, it is easy to create quantum channels by emitting one of the entangled LC modes to a receiver at a long distance (see a schematic plot in Fig. 2). These features make this entangled coherent-state generator unique in the further development of quantum information processing. Finally, the LC circuits coupling to a flux qubit on a chip and the operations and detection of the qubit states require no new technology as far as we can see, as all the essential techniques have already been developed
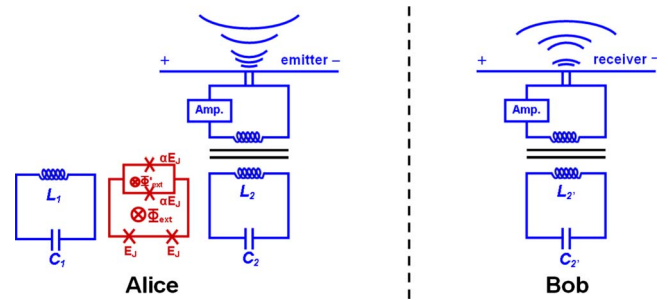

FIG. 2. (Color online) After generating the entangled coherent state of the two LC modes, we can use the antennas to emit one of the two entangled LC modes to a long-distance receiver without using waveguides or fibers.

in various experiments. These advantages increase the feasibility of this entanglement coherent-state generator in practice.

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